Exam I (Part II) **Name**

Please Note: Calculators may be used only in elementary or trig mode, and not in calculus mode (even for exploratory purposes). Please note also that for your solutions to receive full credit they must be complete, precisely formulated, and cast in a well organized, legible way.

The first eight problems are worth 10 points each. Solve them on the extra sheets of paper provided. Then place your answers in the corresponding blanks below. For two of these problems the fact that $\tan\frac{\pi}{4} = 1$ is useful.

1. The derivative of $f(x) = \tan^3 x = \frac{\sin^3 x}{\cos^3 x}$ is equal to ______________________________ .

2. The graph of the function $x^3 + 3x^2 - 9x + 5$ is:

increasing over the interval(s): ____________________

deceasing over the interval(s): ____________________

concave up over the interval(s): ____________________ and

concave down over the interval(s): ____________________ .

3. Let c and d be constants. The function defined by $f(x) = e^{x^2}$ for $x \leq 2$ and $f(x) = c \log_2 x + d$ for $x \geq 2$ is differentiable for all real numbers x for

$c =$ ______________________ and $d =$ ______________________ .

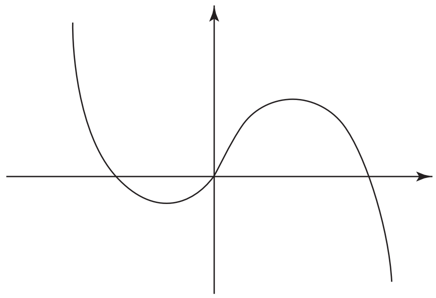
4. Let $f(x) = (\tan^{-1} x)^{\ln x}$. The derivative of $f(x)$ evaluated at $x = 1$ is equal to ________________.

5. Let $f(x) = \sinh^2(\sqrt{x})$. Then $f'(2)$ expressed (or written) as a limit is

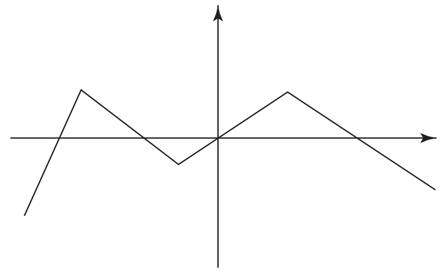
$f'(2) =$ ____

and the value of $f'(2)$ is ____________________________.

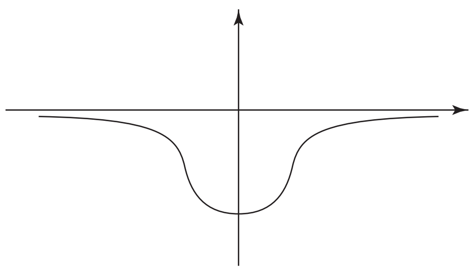
6. Match the graphs of the functions i-iv with the graphs of their derivatives a-d.



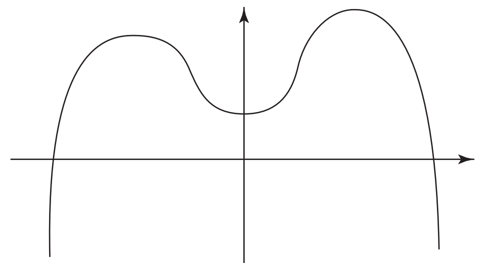
graph i



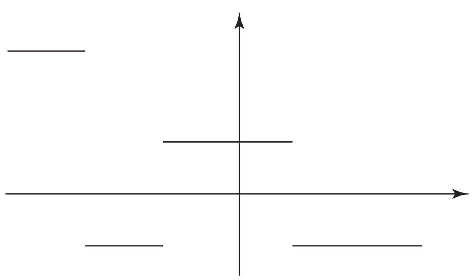
graph ii



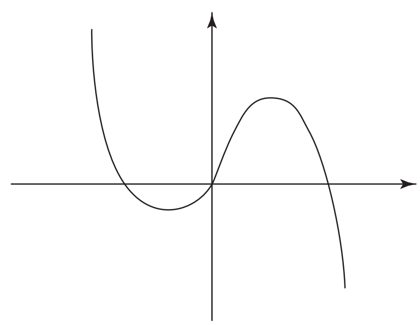
graph iii



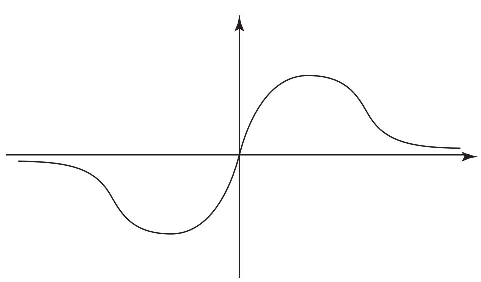
graph iv



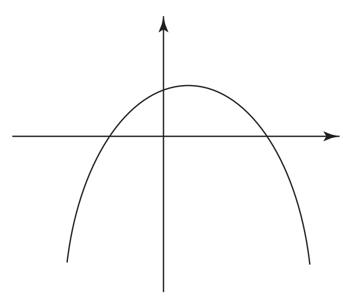
graph a



graph b



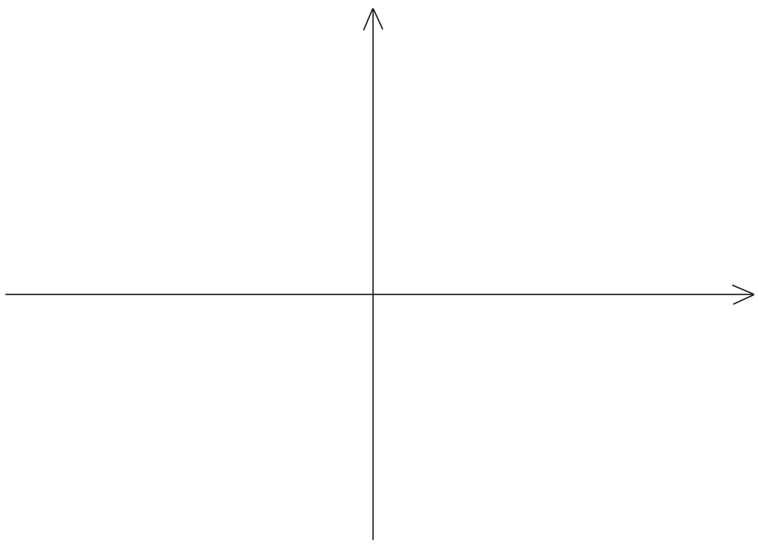
graph c



graph d

Answer: i __________ ii __________ iii __________ iv __________

7. (10 pts) In the space provided below, sketch the graph of a function $f(x)$ with the following properties: $f'(x) > 0$ for $x < -3$, $f'(x) < 0$ for $-3 < x < 0$, $f'(x) > 0$ for $x > 0$; $f''(x) > 0$ for $x < -3$, $f''(x) > 0$ for $-3 < x < -1$, $f''(x) < 0$ for $-1 < x < 0$, and $f''(x) < 0$ for $x > 0$. The lines $x = -3$ and $x = 0$ are vertical asymptotes, and the lines $y = 1$ and $y = -1$ are horizontal asymptotes.

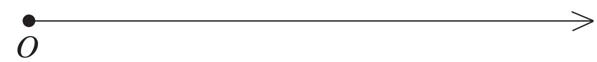


8. Consider a polar and xy-coordinate system simultaneously. The graph of the polar function $f(\theta) = \frac{3}{1+\cos\theta}$ is a parabola with focal point ________ and directrix the line ________.
The equation ____________________________ is a Cartesian equation of this parabola.

Continue to consider the polar function $f(\theta) = \frac{3}{1+\cos\theta}$ and check that the point $P = (3, \frac{\pi}{2})$ is on its graph. The angle that the tangent to the graph of $f(\theta)$ at P makes with the segment OP is ____________.

9. Consider a polar and x-y coordinate system simultaneously. Consider also the polar equation $r = 2\sin\theta + 4\cos\theta$ with $0 \leq \theta \leq 2\pi$.

i. Convert this polar equation into Cartesian form and show that this Cartesian version is the equation of a circle. Then sketch this circle carefully into the coordinate plane below.



ii. By relying only on the graph above compute the integral $\displaystyle\int_0^{\frac{\pi}{2}} \tfrac{1}{2}(2\sin\theta + 4\cos\theta)^2 \, d\theta$.

Formulas and Facts:

$A = \frac{1}{2}r^2\theta$

$a^2 = b^2 + c^2 \quad \varepsilon = \frac{c}{a} \quad A = ab\pi$

$F = ma$, $\kappa = \frac{A_t}{t}$

$M = \frac{4\pi^2 a^3}{GT^2} \qquad G = 6.67 \times 10^{-11}$ in M.K.S. $\qquad F = G\frac{mM}{r^2} \qquad F = \frac{8\kappa^2 m}{L}\frac{1}{r_P^2} \qquad \frac{a^3}{T^2} = \frac{GM}{4\pi^2}.$

$\frac{d}{dx}a^x = \ln a \cdot a^x \ \log_a x = \frac{1}{\ln a} \cdot \ln x$

$\frac{d}{dx}\sin^{-1} x = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}\tan^{-1} x = \frac{1}{x^2+1}$

$x = r\cos\theta, \quad y = r\sin\theta, \quad r^2 = x^2 + y^2, \quad \tan\theta = \frac{y}{x}$

$r(t)^2 \cdot \theta'(t) = c \qquad F(t) = m\left[\frac{4\kappa^2}{r(t)^3} - \frac{d^2 r}{dt^2}\right]$

$x \cdot \frac{dy}{dt} - y \cdot \frac{dx}{dt} \qquad 2\frac{dr}{dt} \cdot \frac{d\theta}{dt} + r \cdot \frac{d^2\theta}{dt^2} = 0$

$\int_a^b \sqrt{1 + f'(x)^2}\, dx \qquad \int_a^b \sqrt{f'(\theta)^2 + f(\theta)^2}\, dx$

A $= \int_a^b \frac{1}{2} f(\theta)^2\, d\theta$

$f'(\theta) = f(\theta) \cdot \tan(\gamma - \frac{\pi}{2})$

$\sinh x = \frac{e^x - e^{-x}}{2} \qquad \cosh x = \frac{e^x + e^{-x}}{2}$